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Proving the PP-wave/CFT_2 duality

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Abstract: We study the duality between IIB string theory on a pp-wave background, arising as a Penrose limit of the $AdS_3 \times S^3 \times M$, where M is T^4 (or $K3$), and the 2D CFT which is given by the $\mathcal{N} = (4,4)$ orbifold $(M)^N/S_N$, resolved by a blowing-up mode. After analizying the action of the supercharges on both sides, we establish a correspondence between the states of the two theories. In particular and for the T^4 case, we identify both massive and massless oscillators on the pp-wave, with certain classes of excited states in the resolved CFT carrying large R-charge n. For the former, the excited states involve fractional modes of the generators of the $\mathcal{N} = 4$ chiral algebra acting on the Z_n ground states. For the latter, they involve, fractional modes of the $\mathrm{U}(1)^4_L \times \mathrm{U}(1)^4_R$ super-current algebra acting on the Z_n ground states. By using conformal perturbation theory we compute the leading order correction to the conformal dimensions of the first class of states, due to the presence of the blowing up mode. We find agreement, to this order, with the corresponding spectrum of massive oscillators on the pp-wave. We also discuss the issue of higher order corrections.

Keywords: AdS-CFT and dS-CFT Correspondence, Penrose limit and pp-wave background, D-branes.

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1. Introduction

The recent discovery of exactly solvable pp-wave string backgrounds as particular double scaling limits of AdS type-IIB backgrounds [1, 2, 3] has made possible to extend the AdS/CFT correspondence beyond the supergravity approximation on the string theory side and therefore, in the dual CFT, beyond the class of strictly protected chiral primary operators. This has been first achieved in the context of the $AdS_5 \times S^5$ / $\mathcal{N} = 4$ duality [4] and then extended to some of its variations [5], including a case involving open strings/fundamental matter [6]. Another interesting dual pair which has been extensively studied in the past [8]–[12] is given by IIB string theory on $AdS_3 \times S^3 \times M$, where M is either T^4 or $K3$ on one side and an $\mathcal{N} = (4, 4)$ two-dimensional CFT correponding to the sigma model M^N/S_N, In the Penrose limit this background gives rise an exactly solvable (in the light-cone gauge, Green-Schwarz formulation) pp-wave [4, 7] string background. Purpose of this paper is to provide some detailed quantitative tests of the duality between string theory on this pp-wave and above 2D CFT. The first step towards establishing the duality is of course to provide a correspondence between the states of the two theories: on the string theory side we have the ground state(s) and the states built with massive and massless (for T^4 case) oscillators, including zero modes. We will propose a precise correspondence, guided by the action of supersymmetry on the states on the two theories. On the CFT side the corresponding class of states is characterized large R-charge J, $J \sim \sqrt{N}$, and J itself is identified with the n of Z_n twisted sectors.

After identifying ground states with chiral primary states of the CFT and massless oscillator modes with first level descendants of the left-moving $\mathcal{N} = 4$ algebra generators L_1, $G^-_{-\frac{1}{2}}$, J^-_0, (and similarly for the right-moving one, $\tilde{L}_1$, $\tilde{G}^-_{-\frac{1}{2}}$, $\tilde{J}^-_0$) we move on to massive oscillators, which will be identified with certain, say, left fractionally moded descendants, with generators $L_{-1+\frac{k}{J}}$, $G^-_{-\frac{1}{2}+\frac{k}{J}}$, $J^-_{-\frac{k}{J}}$, with $k \ll J$, acting on the Z_J ground states. These

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states are right-chiral at the orbifold point, but become non-chiral when the orbifold is resolved by switching on a Z_2 blowing up mode. The necessity of moving off the orbifold point or, in other words, the fact that the point in moduli space where the CFT is dual to the pp-wave is not the orbifold point, is suggested by the spectrum of the light-cone hamiltonian, p^-, to be identified in the CFT with $\Delta - J$, Δ being the conformal dimension of the state. The spectrum is composed of two contributions. Let us consider the first one, which is due to the massive oscillators from the pp-wave part of the background, and, therefore, has the typical square-root form, implying that the conformal dimensions of the corresponding states admit an expansion in powers of $g_6^2 N/J^2$, where g_6 is the 6-dimensional string coupling, much like what happens in the Yang-Mills case [4]. We will see that switching on the blowing-up mode has the effect of lifting the class of states in consideration, which are right-chiral (and therefore belong to short multiplets) at the orbifold point and of making four of them to join into long multiplets. As a condequence, their conformal dimensions are unprotected and are expected to receive corrections. We will verify this fact by showing that the (right-moving) supercharges are indeed modified in presence of the blowing up mode.

The occurrence of this phenomenon will be also supported by a partition function computation on the two theories, which, as we will see, also suggests the presence of a supercharge with a Z_2 twist nature, in agreement with the explicit CFT argument. We will perform a CFT calculation of the leading order change in the conformal dimension, finding agreement with the first order expansion of the square root formula from the pp-wave side.

The second term in the p^- hamiltonian is due, in the T^4 case, to the usual massless oscillators (we do not consider momenta and windings along T^4). This contribution is proportional to $1/J$. We propose that on the CFT side these correspond to fractional modes of the $\mathrm{U}(1)_L^4 \times \mathrm{U}(1)_R^4$ super-current algebra acting on the ground states on both left- and right-moving sectors, subject to the level matching condition. First order CFT perturbation theory gives, also for these states, results in qualitative agreement with the predictions from string theory spectrum.

The paper is organized as follows. In section 2, we briefly review AdS_3/CFT_2 duality and its Penrose limit. The supersymmetry generators of the pp-wave limit of the $\mathcal{N}(4,4)$ algebra are constructed in section 3. In particular we find that the anti-commutator of the left and right supercharges is proportional to the world-sheet momentum operator P_σ and therefore vanishes on the physical states. In section 4, we present the computation of the first order corrections to the conformal dimensions in the boundary CFT. Finally we discuss the question of higher order corrections and, assuming a natural identification between the states on the two sides, we show that the exact expression for the conformal dimensions imply an extension of the superalgebra: the anticommutator between the left and right supercharges vanishes on the orbifold group invariant states but not on individual fractional oscillators. This is exactly analogous to the results obtained in section 3 for the pp-wave strings.

While this paper was in progress, three papers appeared, [13, 20] and [15] addressing some of the issues discussed here and overlapping partially with ours.

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2. AdS_3/CFT_2 duality and its Penrose limit

In this section we review the basic facts about the duality between IIB string theory on $AdS_3 \times S^3 \times M$ and the two dimensional boundary CFT corresponding to the symmetric product M^N/S_N. On the string theory side, the above background arises as near horizon geometry of a system of Q_1 D1-branes and Q_5 parallel D5-branes wrapped on M, in the limit where both Q_1 and Q_5 are large at fixed Q_1/Q_5. The radius R of both AdS_3 and S^3 is given by $R^2 = \alpha' g_6 \sqrt{N}$, with $N = Q_1 Q_5$. g_6 is the six dimensional effective coupling given in terms of the IIB string coupling g_s as $g_6 = g_s \sqrt{Q_5/Q_1}$. String theory on such background is believed to be dual to certain two dimensional $\mathcal{N} = (4,4)$ SCFT in the Neveu-Schwarz sector. The CFT corresponds to the orbifold sigma model M^N/S_N, possibly blown-up, depending on the values of spacetime moduli which survive the near horizon limit.

There have been several tests of this duality, mainly at the level of spectrum of protected operators [9]–[12]. In particular, as first done in [10], the elliptic genus of the CFT has been compared with the one, suitably defined, of supergravity. The two have been shown to agree for low energy excitations [10, 11, 12]. Notice that, in this comparison, one is including also multiparticle supergravity states, but our discussion here will be focused only on supergravity (string theory) single particle states. For our later purposes, it will be useful to reconsider such computations. The elliptic genus involves states of the form (anything, chiral), but actually it does not keep track of the J^3_R R-quantum numbers of the right-moving Ramond ground states, since one sets the conjugate variable $\tilde{y}$ to $\bar{q}^{-1/2}$ in the character valued partition function,[1] $\mathrm{tr}_{NS}(-)^F q^{L_0 - c/24} y^{J^3_L}$. One may ask then what happens if we keep arbitrary $\tilde{y}$ for the chiral states in the partition function. In general one does not expect the resulting partition function to be a topological quantity, and indeed it was already observed in [12] that in this case there is a discrepancy between supergravity and CFT results. We want to analyze this point in more detail. From the CFT side, the partition function for M^N/S_N is best obtained via a generating function $Z_{CFT} = \sum_N p^N Z(M^N/S_N)$ which can be easily computed once the multiplicities for the single copy CFT, $c_{CFT}(m, \ell, \bar{m}, \tilde{\ell})$, are given. Taking into account only states which correspond to single particle supergravity states, i.e. the states coming from the twisted sectors corresponding to single cycles, amounts to consider

$$Z_{\mathrm{CFT}}(p, q, \bar{q}, y, \tilde{y}) = \sum c_{\mathrm{CFT}} \left(4m - n^2 - \ell^2, 4\bar{m} - n^2 - \tilde{\ell}^2\right) p^n q^m y^\ell \bar{q}^{\bar{m}} \tilde{y}^{\tilde{\ell}} \tag{2.1}$$

where the multiplicities c_{CFT}'s are obtained from the single copy CFT partition function. The single particle partition function for states of the form (anything, chiral) on the supergravity side is computed via the prescription of [10], once the spectrum of chiral primaries is known (which happens to coincide with the cohomology of M), and has an expansion of the form:[2]

$$Z_{\mathrm{sugra}} = \sum_{n,m,\ell,\tilde{\ell}} c_{\mathrm{sugra}}(n, m, \ell) p^n q^m y^\ell \left(\tilde{y} \bar{q}^{1/2}\right)^{\tilde{\ell}}. \tag{2.2}$$

[1] The elliptic genus is defined in the Ramond sector, where one usually sets $\tilde{y} = 1$, but we need to flow to the NS sector, for the reason explained before.

[2] The crucial prescription in this derivation has to do with the notion of *degree* formulated in [10], allowing to discuss finite N in supergravity, and therefore to introduce the variable p.

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Let us recall that the agreement between elliptic genera for states of low conformal dimensions ($m \leq N/4$) amounts to the equality $Z_{\rm CFT}(p=1,q,y) = Z_{\rm sugra}(p=1.q,y)$ and $\frac{\partial}{\partial p} Z_{\rm CFT}(p,q,y)|_{p=1} = \frac{\partial}{\partial p} Z_{\rm sugra}(p.q,y)|_{p=1}$, that is, the two elliptic genera differ by an expression of the form $(p-1)^2 g(p,q,y)$. In the above discussion we have set $\tilde{y} = \bar{q}^{-1/2}$, which is the value corresponding to the elliptic genus flowed in the NS sector. However, using the techniques explained in [10, 12], one can repeat the analysis for the case where one considers right- moving chiral states with the appropriate power of $\tilde{y}$ given by their R-charge. We will not repeat here the calculation, but what one gets is that the CFT and supergravity partition functions differ by an expression of the form $(1 - p\bar{q}^{1/2}\tilde{y})^2 f(p,q,y,\bar{q}^{1/2}\tilde{y})$. This suggests that what can happen in the CFT is that four short multiplets, coming from the sectors n, (1), $n+1$, (2) and $n+2$, (1) respectively can join into a long multiplet and drop from the elliptic genus. In other words, one may expect that, as we move in the CFT moduli space, the Higgs mechanism can take place and assemble short multiplets (whose number is a multiple of four) into long ones. We will explicitly prove in the next section that this is indeed what happens, and we will identify these states with a class of (massive) string oscillator states on the pp-wave background.

Let us consider now the Penrose limit of the $AdS_3 \times S^3 \times M$ background [4, 15]. This is obtained by blowing up a region near a null geodesic in $AdS_3 \times S^3$. It involves a scaling limit where $R \to \infty$ with α', g_s and g_6 finite. The resulting metric becomes

$$ds^2 = -2dx^+dx^- - \mu^2 \left(x^2 + y^2\right) dx^+dx^- + d\vec{x}^2 + d\vec{y}^2 + ds_M^2 \tag{2.3}$$

where $\vec{x}$ and $\vec{y}$ are two- dimensional vectors, parametrizing a four-dimensional transverse flat space. There are in addition non trivial vev's for the RR 3-form field strength, $H_{+12} = H_{+34} = \mu$.

The light-cone momenta in this background, $p^+ = i\partial_{x^-}$ and $p^- = i\partial_{x^+}$, where p^- is the light-cone hamiltonian, are identified with the following combinations of charges in the CFT:

$$\begin{aligned} p^- &= \mu(\Delta - J) \\ p^+ &= J/\mu R^2 \end{aligned} \tag{2.4}$$

where Δ is the total conformal dimension in the CFT and J is the U(1) R-symmetry charge $J = J_L^3 + J_R^3$, with $J_{L,R}^3$ Cartan currents in the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ R-symmetry algebra of the $\mathcal{N} = (4,4)$ SCFT. Unitarity in the CFT gives $\Delta - J \geq 0$, corresponding to non-negativity of the light-cone hamiltonian. The ground states of the latter correspond to chiral primary states in the CFT. The requirement of considering finite energy excitations in the limit $R \to \infty$ (i.e. $N \to \infty$), demands to take $J \sim \sqrt{N}$ with $\Delta - J$ finite.

The spectrum of the light-cone hamiltonian is found to be [4, 7]

$$p^- = \sum_n N_n \sqrt{1 + \left(\frac{n}{\mu p^+ \alpha'}\right)^2} + \frac{L_0^M + \tilde{L}_0^M}{\mu p^+ \alpha'} \tag{2.5}$$

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with the level-matching constraint on the occupation numbers N_n:

$$\sum_n N_n = L_0^M - \tilde{L}_0^M \tag{2.6}$$

where L_0^M and $\tilde{L}_0^M$ are left- and right-moving Virasoro generators for the CFT corresponding to M.

With the identifications given above, in particular $\mu p^+ \alpha' = J/g_s Q_5$, we can translate the pp-wave spectrum (2.5) into the following statement for the CFT spectrum:

$$\Delta - J = \sum_n N_n \sqrt{1 + \left(\frac{n g_s Q_5}{J}\right)^2} + g_s Q_5 \frac{L_0^M + \tilde{L}_0^M}{J} \,. \tag{2.7}$$

Our main goal will be to reproduce (2.7) directly from the CFT.

3. Symmetry Generators

We now identify the $\mathcal{N} = (4,4)$ generators on the two sides, namely string theory on the pp-wave and two-dimensional CFT.

In the pp-wave background, the mass terms break the light-cone SO(8) symmetry down to $\mathrm{SO}(2) \times \mathrm{SO}(2)' \times \mathrm{SO}(4)$ where the first and second SO(2) factors act on the directions coming from AdS_3 (say directions X^1 and X^2) and S^3 (say X^3 and X^4) respectively, while the SO(4) acts on the tangent space of T^4 (or K_3). The generators of the first and the second SO(2) therefore should be identified with $L_0 - \tilde{L}_0$ and $\tilde{j}_0 - j_0$ respectively. The total dimension Δ and charge J are $\Delta = L_0 + \tilde{L}_0$ and $J = \tilde{j}_0 + j_0$. The fields $X^I, I = 1, .., 4$ are massive, while the directions along the T^4, which we denote by Y^i, are massless. The two SO(8) spinors decompose into $\theta^i_{\alpha a}$ and $\chi^i_{\dot\alpha \dot a}$ where $i = 1, 2$ denotes the worldsheet chirality α and $\dot\alpha$ denote the two components of the first SO(4) (which breaks down to $\mathrm{SO}(2) \times \mathrm{SO}(2)'$ due to the fermion mass term) spinor and anti-spinor respectively. a and $\dot a$ denote the two components of the spinor and anti-spinor of the second SO(4), acting on the tangent space of T^4. These fermions satisfy the reality conditions $\bar\theta^i_{\alpha a} = \epsilon^{\alpha\beta}\epsilon^{ab}\theta^i_{\beta b}$ and similarly $\bar\chi^i_{\dot\alpha \dot a} = \epsilon^{\dot\alpha\dot\beta}\epsilon^{\dot a \dot b}\chi^i_{\dot\beta \dot b}$. While θ are massive, with mass being the same as that of X^I, χ are massless. The equations of motion for the massive fields are:

$$\partial_+\partial_- X^I = (\mu p^+)^2 X^I \,, \quad \partial_+ \theta^1_{\alpha a} - i\mu p^+ (\sigma^3)_\alpha{}^\beta \theta^2_{\beta a} = 0 \,, \quad \partial_- \theta^2_{\alpha a} + i\mu p^+ (\sigma^3)_\alpha{}^\beta \theta^1_{\beta a} = 0 \,. \tag{3.1}$$

Here and in the following we set $\alpha' = 1/2\pi$. The conserved supercharges have the following expression:

$$\begin{aligned}
Q^c_{\alpha a} &= \sqrt{p^+} \int d\sigma \left(e^{i\mu X^+ \sigma_3}\right)_\alpha^\beta \theta^c_{\beta a} \,, \qquad c = 1, 2 \\
Q^{1\dot\alpha}_a &= \int d\sigma \left(\partial_- X^\mu \bar\sigma^{\mu \dot\alpha\beta} \theta^1_{\beta a} - i\mu p^+ X^\mu \bar\sigma^{\mu\dot\alpha\beta} (\sigma_3 \theta^2)_{\beta a} + \partial_- Y^i \sigma^i_{a\dot b} \chi^{1\dot\alpha\dot b}\right) \\
Q^{2\dot\alpha}_a &= \int d\sigma \left(\partial_- X^\mu \bar\sigma^{\mu \dot\alpha\beta} \theta^2_{\beta a} - i\mu p^+ X^\mu \bar\sigma^{\mu\dot\alpha\beta} (\sigma_3 \theta^1)_{\beta a} + \partial_- Y^i \sigma^i_{a\dot b} \chi^{2\dot\alpha\dot b}\right)
\end{aligned} \tag{3.2}$$

c in the first equation above denotes the two world-sheet chiralities and $(\sigma_3\theta)_{\alpha a}$ denotes $(\sigma_3)_\alpha{}^\beta \theta_{\beta a}$.

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The mode expansions for the massless fields Y^i and χ are standard, with the clear separation of left and right movers. The zero modes of χ generate the cohomology of T^4: the two complex left-moving zero modes and two complex right-moving zero modes acting on the ground state reproduce the cohomology elements $h_{p,q}$ of T^4.

For later convenience it is better to organize the fields in terms of definite $\mathrm{SO}(2)\times\mathrm{SO}(2)'$ charges. Thus we define two complex bosons $Z^1 = X^1 + iX^2$ and $Z^2 = X^3 + iX^4$. The complex conjugate fields will be denoted as $Z^{\bar{i}}$. the mode expansion for the massive fields Z^I and θ are

$$\begin{aligned}
Z^i(\sigma,\tau) &= i\sum_k \frac{1}{\sqrt{2}\omega_k}\left(e^{-i\omega_k\tau+ik\sigma}a_k^i - e^{i\omega_k\tau-ik\sigma}a_k^{\bar{i}\dagger}\right), \qquad i=1,2\\
\theta^1_{\alpha a}(\sigma,\tau) &= \left[\sum_{k\geq 0} c_k e^{-i\omega_k\tau}\left(e^{ik\sigma}(b_k)_{\alpha a} + \frac{\omega_k - k}{\mu p^+}e^{-ik\sigma}(b_{-k})_{\alpha a}\right)\right] +\\
&\quad +\epsilon_{\alpha\beta}\epsilon_{ab}\sum_{k\geq 0} c_k e^{i\omega_k\tau}\left(e^{-ik\sigma}\left(b_k^\dagger\right)^{\beta b} + \frac{\omega_k - k}{\mu p^+}e^{ik\sigma}\left(b_{-k}^\dagger\right)^{\beta b}\right)\\
\theta^2_{\alpha a}(\sigma,\tau) &= -\left[\sum_{k\geq 0} c_k e^{-i\omega_k\tau}\left(e^{ik\sigma}\left(\sigma_3 b_k\right)_{\alpha a} + \frac{\omega_k - k}{\mu p^+}e^{-ik\sigma}\left(\sigma_3 b_{-k}\right)_{\alpha a}\right)\right] +\\
&\quad +\epsilon_{\alpha\beta}\epsilon_{ab}\sum_{k\geq 0} c_k e^{i\omega_k\tau}\left(e^{-ik\sigma}\left(\sigma_3 b_k^\dagger\right)^{\beta b} + \frac{\omega_k - k}{\mu p^+}e^{ik\sigma}\left(\sigma_3 b_{-k}^\dagger\right)^{\beta b}\right)
\end{aligned} \tag{3.3}$$

where

$$\omega_k = \sqrt{k^2 + (\mu p^+)^2}\,, \tag{3.4}$$

and the normalization constant

$$c_k = \frac{\mu p^+}{\sqrt{2\omega_k(\omega_k - k)}}\,. \tag{3.5}$$

The oscillator modes satisfy the usual (anti-) commutation relations

$$[a_k^i, a_\ell^{\bar{j}\dagger}] = [a_k^{\bar{i}}, a_\ell^{j\dagger}] = \delta_{ij}\delta_{k\ell}\,, \qquad \{b_{k\alpha a}, b_\ell^{\dagger\beta b}\} = \delta_\alpha^\beta\delta_a^b\delta_{k\ell}\,. \tag{3.6}$$

Now we are ready to identify the symmetry generators of $\mathcal{N}=(4,4)$ algebra. The left and right supersymmetry generators are:

$$\begin{aligned}
G^{+a}_{1/2} &= \sqrt{\mu p^+}\,R(b_0)^a_2\,, \qquad \tilde{G}^{+a}_{1/2} = \sqrt{\mu p^+}\,R(b_0)^a_1\,,\\
G^{-a}_{-1/2} &= \sqrt{\mu p^+}\,R\left(b_0^\dagger\right)^{2a}\,, \qquad \tilde{G}^{-a}_{-1/2} = \sqrt{\mu p^+}\,R\left(b_0^\dagger\right)^{1a}\,,\\
G^{-a}_{1/2} &= \frac{1}{\sqrt{p^+}}\left((Q^1)^{\dot{1}a} + (Q^2)^{\dot{1}a}\right), \qquad \tilde{G}^{-a}_{1/2} = \frac{1}{\sqrt{p^+}}\left((Q^1)^{\dot{1}a} - (Q^2)^{\dot{1}a}\right),\\
G^{+a}_{-1/2} &= \frac{1}{\sqrt{p^+}}\left((Q^1)^{\dot{2}a} + (Q^2)^{\dot{2}a}\right), \qquad \tilde{G}^{+a}_{-1/2} = \frac{1}{\sqrt{p^+}}\left((Q^1)^{\dot{2}a} - (Q^2)^{\dot{2}a}\right).
\end{aligned} \tag{3.7}$$

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The bosonic left and right SL$(2,R)$ and SU(2) generators are:

$$\begin{aligned} L_1 &= \sqrt{\mu p^+} R a_0^2, & L_{-1} &= \sqrt{\mu p^+} R a_0^{\bar{2}\dagger}, \\ \tilde{L}_1 &= \sqrt{\mu p^+} R a_0^{\bar{2}}, & \tilde{L}_{-1} &= \sqrt{\mu p^+} R a_0^{2\dagger}, \\ J_0^+ &= \sqrt{\mu p^+} R a_0^{\bar{1}}, & J_0^- &= \sqrt{\mu p^+} R a_0^{1\dagger}, \\ \tilde{J}_0^+ &= \sqrt{\mu p^+} R a_0^1, & \tilde{J}_0^- &= \sqrt{\mu p^+} R a_0^{\bar{1}\dagger}. \end{aligned} \tag{3.8}$$

For later purposes we give an explicit expression for the part of $\tilde{G}$ that depends on massive fields:

$$\begin{aligned} \tilde{G}^{+a}_{-1/2} &= \frac{1}{\sqrt{2p^+}} \sum_k \Bigg[\sqrt{\omega_k + p^+} \left(a_k^{2\dagger} (b_k)_1^a + a_k^1 \left(b_k^\dagger \right)^{1a} \right) + \\ &\quad + \mathrm{sign}(k) \sqrt{\omega_k - p^+} \left(a_k^{1\dagger} (b_k)_2^a + a_k^2 \left(b_k^\dagger \right)^{2a} \right) \Bigg] \\ \tilde{G}^{-a}_{1/2} &= \frac{1}{\sqrt{2p^+}} \sum_k \Bigg[\sqrt{\omega_k + p^+} \left(a_k^{\bar{1}\dagger} (b_k)_1{}^a - a_k^{\bar{2}} \left(b_k^\dagger \right)^{1a} \right) + \\ &\quad + \mathrm{sign}(k) \sqrt{\omega_k - p^+} \left(a_k^{\bar{2}\dagger} (b_k)_2{}^a - a_k^{\bar{1}} \left(b_k^\dagger \right)^{2a} \right) \Bigg]. \end{aligned} \tag{3.9}$$

The expressions for $G^{\pm a}_{\mp 1/2}$ is the same as that of $\tilde{G}^{\mp a}_{\pm 1/2}$, with $\sqrt{\omega_k + p^+}$ exchanged by $\sqrt{\omega_k - p^+}\mathrm{sign}(k)$. One can verify that the anti-commutation relations between these G's are correct. In particular, the anticommutator $\{G^-_{1/2}, G^+_{-1/2}\}$ defines $L_0 - j_0$ and similarly for the tilde operators. As a consistency check, one finds that

$$\left\{ G^{-a}_{1/2}, G^{+b}_{-1/2} \right\} + \left\{ \tilde{G}^{-a}_{1/2}, \tilde{G}^{+b}_{-1/2} \right\} = \epsilon^{ab} \left(L_0 - j_0 + \tilde{L}_0 - \tilde{j}_0 \right) = \epsilon^{ab} (\Delta - J) = \epsilon^{ab} P^- \tag{3.10}$$

where, as in the following, we set $\mu = 1$. Note that while $G^{\pm}_{\mp 1/2}$ anticommutes with $\tilde{G}^{\mp}_{\pm 1/2}$ as they should, with $\tilde{G}^{\pm}_{\mp 1/2}$ they anti-commute only on physical states in the pp-wave string theory, i.e. the states that satisfy the level matching condition. Indeed we find

$$\left\{ G^{-a}_{1/2}, \tilde{G}^{-b}_{1/2} \right\} = \epsilon^{ab} \frac{1}{2p^+} \sum_k k N_k = \epsilon^{ab} \frac{1}{2p^+} P_\sigma \tag{3.11}$$

where N_k is the oscillator number operator at momentum k on the pp wave side and P_σ just denotes the total momentum. This might seem strange, but does not create any problem since on the physical states these anti-commutators vanish. We will see in section 5, the analog of the pp-wave physical state condition as well as the analog of this anti-commutator in the dual perturbed boundary CFT.

From this identification it is clear that the insertion of zero mode creation operators of the massive fields on the pp-wave side correspond to the insertions of L_{-1}, $\tilde{L}_{-1}$, J_0^-, $\tilde{J}_0^-$ (bosonic) and $G^{-\pm}_{-1/2}$, $\tilde{G}^{-\pm}_{-1/2}$ on the boundary CFT side.

How about the non-zero modes on the pp-wave side ? It is natural to identify them with the insertions of fractional modes of $L_{-1+k/n}$, $\tilde{L}_{-1+k/n}$, $J^-_{k/n}$, $\tilde{J}^-_{k/n}$ (bosonic) and

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$G^{-\pm}_{-1/2+k/n}$, $\tilde{G}^{-\pm}_{-1/2+k/n}$ acting on the Z_n twisted sector of the boundary CFT (here k can be positive or negative but is assumed that $|k| \ll n$.[3])

However, one immediately comes up against an apparent contradiction to this proposal. Consider, for instance, a state in the boundary CFT of the form $J^-_{k/n}G^{-b}_{-1/2-k/n}|n\rangle \equiv \Psi_1$, where $|n\rangle$ denotes the chiral-chiral ground state in the Z_n twisted sector with left and right charges being $(n-1)/2$. This state is non-chiral in the left-moving sector but is chiral on the right-moving sector. It therefore must be in a (intermediate) BPS multiplet. In other words, $\tilde{G}^{+a}_{-1/2}$ annihilates this state. On the other hand, according to our proposal, this state is to be identified with $a^{1\dagger}_{-k}(b^\dagger)^{2b}_k|p^+\rangle$, where p^+ is essentially n. It is easy to see, from the explicit expression of $\tilde{G}^{+a}_{-1/2}$ given in (3.9), that it does not annihilate this state. In fact it transforms the state into $d(k)a^{1\dagger}_{-k}a^{1\dagger}_k|p^+\rangle$, where the coefficient $d(k) = \mathrm{sign}(k)\sqrt{\frac{\omega_k - p^+}{2p^+}}$ is an odd function of k and in the large p^+ limit reduces to k/p^+. This state, according to our dictionary, is proportional to the state $J^-_{k/n'}J^-_{-k/n'}|n'\rangle \equiv \Psi_2$ in the boundary CFT for some n'. By examining the charges we can conclude that $n' = n+1$. In any case this implies that if the dictionary between boundary CFT and pp-wave string is correct then such states on boundary CFT side must become non-chiral via some Higgs mechanism, i.e. several (4 in this case) intermediate multiplets should join together to give a long multiplet. The fact that the Z_n twisted sector is mapped to the Z_{n+1} twisted state suggests that this Higgs mechanism is provided by the exactly marginal Z_2 twist field acquiring a non-vanishing expectation value.

Another way to see this problem is that the state Ψ_1 in the boundary CFT has $\Delta - J = 2$ while the corresponding state on the pp-wave side has $\Delta - J = 2\sqrt{1+k^2/(p^+)^2} \sim 2 + k^2/(p^+)^2 + \ldots$ Since $\tilde{L}_0 - \tilde{j} = \{\tilde{G}^{-a}_{+1/2}\tilde{G}^{+b}_{-1/2}\}$, the change in the dimension can be obtained by computing the change in $\tilde{G}$ under the marginal deformation. In particular, since the change in the dimension is not zero, the state cannot be a chiral primary. In the next section we will compute the first order change in $\tilde{G}^{+a}_{-1/2}$ due to the marginal deformation and show that the state Ψ_1 indeed is mapped to state Ψ_2 with the coefficient $d(k)$.

Assuming that the identification between the pp-wave string states and the boundary CFT states is correct, we can then summarize the action of $\tilde{G}$ on the various left moving excitations, in the large J limit, as follows:

$$\begin{aligned}
\tilde{G}^{+a}_{-1/2} : G^{-b}_{-1/2+k/n}|n\rangle &\to \frac{k}{\sqrt{2}p^+}\epsilon^{ab}J^-_{k/(n+1)}|n+1\rangle\,, \\
L_{-1+k/n}|n\rangle &\to \frac{k}{\sqrt{2}p^+}G^{-a}_{-1/2+k/(n+1)}|n+1\rangle\,, \\
\tilde{G}^{-a}_{1/2} : G^{-b}_{-1/2+k/n}|n\rangle &\to \frac{k}{\sqrt{2}p^+}\epsilon^{ab}L_{-1+k/(n-1)}|n-1\rangle\,, \\
J^-_{k/n}|n\rangle &\to \frac{k}{\sqrt{2}p^+}G^{-a}_{-1/2+k/(n-1)}|n-1\rangle\,,
\end{aligned} \tag{3.12}$$

[3] Note that $J^-_{k/n}$ for positive but small k are still creation operators acting on the Z_n twisted chiral state which carries a positive charge equal to $(n-1)/2$, $n/2$ or $(n+1)/2$ corresponding to different cohomologies of the T^4. The different cohomology elements are obtained by applying the massless fermion zero modes on the pp wave side

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where $p^+ = J/g_6 \sim n/g_6$. The action of $\tilde{G}$ on the right moving fields is the usual in the large J limit. Note that the third and fourth lines in the above equation are just the hermitean conjugate of the first and second lines.

Another question that comes up with this identification is that, while the massive non-zero mode creation operators commute among themselves and thereby ordering of these operators is irrelevant in the definition of the states, on the boundary CFT side, the corresponding operators do not commute among themselves. While L_{-1}, $G^{-a}_{-1/2}$ and J^-_0 commute among themselves (which correspond to the zero modes on the pp-wave side), the fractional moded operators do not commute. Thus it would appear that different orderings of these operators will give rise to different states. The point, however is that the difference between different orderings vanishes in the sense of norm and inner products, in the large J limit. Consider for example the states $(\frac{1}{n}L_{-1+k/n}J^- - k/n)|n\rangle$ and $(\frac{1}{n}J^- - k/nL_{-1+k/n})|n\rangle$. The factor $1/n$ is just included to have properly normalized states (in the large J limit). The difference between these two states is $\frac{k}{n^2}J^-_{-2}|n\rangle$, whose inner product with either of the above states is of order $1/n^2$ and the norm is of order $1/n^3$. Thus in the large J (or equivalently large-N) limit all the different orderings can be identified with a single state.

Finally, we turn to the pp-wave string states made up of modes of massless fields. As mentioned earlier, the two SO(2)s acting on the complex fields X^2 and X^1 respectively are identified with $L_0 - \tilde{L}_0$ and $\tilde{j} - j$ respectively. This is also clear from equation (3.8). The bosonic massless fields are neutral with respect to the two SO(2)s and therefore the states created by them must carry $L_0 = \tilde{L}_0$ and $j = \tilde{j}$. The fermionic massless field $\chi^{\dot{\alpha}\dot{a}}$ are anti-chiral with respect to the SO(4) which breaks down to the product of these two SO(2)s and therefore carry opposite quantum numbers with respect to the two SO(2)s. Thus states created by these operators must have $L_0 - j = \tilde{L}_0 - \tilde{j}$. Since $L_0 - j$ measures how far the state is from being chiral, such states, on the boundary CFT side, must be equally non-chiral on both left and right moving sectors. It is natural to conjecture that the positive and negative momentum k modes of the T^4 (or $K3$) operators on the pp-wave side (i.e. left and right moving modes on the pp-wave side) should be mapped to states obtained by applying left and right moving k/n fractional moded T^4 (or $K3$) operators in the Z_n twisted sector on the boundary CFT side. The creation operators (for simplicity we consider here T^4 or in case of $K3$ its orbifold limit) acting on Z_n twisted chiral-chiral primary state are the left moving fermionic operators $\psi^{+a}_{-1/2-k/n}$ and $\psi^{-a}_{1/2-k/n}$ (for $a = \pm$) and the bosonic operators $\alpha^i_{-k/n}$ ($i = 1, \ldots, 4$) with $k\rangle 0$ and similarly for the right movers. On the pp wave side the light cone hamiltonian $\Delta - J$ is proportional to k/p^+. This means that on the boundary CFT side we must show that even after perturbation the dimension is proportional to k/n. In the following we study the first order correction to supergenerators and show that to this order it gives a structure constant proportional to $\sqrt{k/n}$.

4. Correlators in the resolved $(T^4)^N/S_N$ CFT and first order correction to the conformal dimensions

As anticipated in the previous section, we need to consider correlation functions in the symmetric product CFT, perturbed by the marginal deformation corresponding to the

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blowing up mode, which resolves the Z_2 orbifold singularity. For earlier work on the symmetric product CFT see [16]–[21].

Although some of the considerations below apply both to the $(K3)^N/S_N$ and $(T^4)^N/S_N$ cases, the explicit computations will be performed for the latter case. The first question we want to address is the modification of the supersymmetry generators when the blowing up mode is turned on. We will discuss this change to first order in the marginal perturbation, following an argument given in [22] for the case of the Virasoro generators.

Recall that the blowing up mode is obtained from the chiral primary field $\sigma_{\frac{1}{2},\frac{1}{2}}$ in the Z_2 twisted sector, of left- right- dimensions (1/2,1/2), by applying the left- and right-moving supercharges $G^{-a}_{-\frac{1}{2}}$ and $\tilde{G}^{-a}_{-\frac{1}{2}}$, where $a = +,-$ denotes the doublet components of a global $\mathrm{SU}(2)_I$ which is an outer automorphism of the $\mathcal{N} = (4,4)$ algebra. Together with the conjugate charges $G^{+a}_{-\frac{1}{2}}$, $\tilde{G}^{+a}_{-\frac{1}{2}}$ they are part of the global $\mathcal{N} = (4,4)$ symmetry of the CFT. Here the explicit $+$ and $-$ refer to the components of a doublet with respect to the global left- and right $\mathrm{SU}(2)_R \times \tilde{\mathrm{SU}}(2)_R$ R-symmetry charges, which are also part of the $\mathcal{N} = (4,4)$ algebra. Therefore there are actually four blowing up modes, which are top components of a short (4,4) multiplet and are neutral under the $\mathrm{SU}(2)_R$, $\tilde{\mathrm{SU}}(2)_R$ Cartan generators, but transform as $\mathbf{3}+\mathbf{1}$ of $\mathrm{SU}(2)_I$. We will be interested in perturbing the symmetric product CFT with the singlet component, corresponding to a particular combination of RR zero- and four-form vev's in the type-IIB background. This is given by an antisymmetric combination of left and right supercharges acting on the chiral primary field, so that the complete expression for the perturbation is:

$$\lambda \left(G^{-+}_{-\frac{1}{2}} \tilde{G}^{--}_{-\frac{1}{2}} - G^{--}_{-\frac{1}{2}} \tilde{G}^{-+}_{-\frac{1}{2}} \right) \sigma_{\frac{1}{2},\frac{1}{2}} + \text{a.c.}\,, \tag{4.1}$$

where λ is the coupling constant, a.c. refers to the expression involving the antichiral field $\bar{\sigma}_{\frac{1}{2},\frac{1}{2}}$ with the hermitean conjugate supercurrents $G^{+\pm}$, $\tilde{G}^{+\pm}$.

We will be interested in studying the action of the right-moving charges $\tilde{G}^{+a}_{-\frac{1}{2}}$, on states (or fields) which are chiral primary on the right-moving sector and excited on the left-moving sector. By definition, $\tilde{G}^{+a}_{-\frac{1}{2}}$ annihilates such states if the blowing up modes are switched off. So let us consider bringing down in a correlator involving these fields the blowing up mode of (4.1) to first order, and look at the action of $\tilde{G}^{+a}_{-\frac{1}{2}}$: this is represented by a contour integral $\oint_C d\bar{z}\tilde{G}^{+a}(\bar{z})$ where the contour C encircles the positions of all the other fields, including the perturbation which is integrated over the plane:

$$\langle \oint_C d\bar{z}\tilde{G}^{+a}(\bar{z}) \prod_i \phi_i(z_i,\bar{z}_i) \int d^2x \sigma_{1,1}(x,\bar{x}) \rangle \tag{4.2}$$

where $\sigma_{1,1} = \epsilon_{ab} G^{-a}_{-\frac{1}{2}} \tilde{G}^{-b}_{-\frac{1}{2}} \sigma_{\frac{1}{2},\frac{1}{2}}$ is the blowing up mode. Deforming the contour around z_i's gives zero if the ϕ_i are right-chiral. The integral around x can be evaluated using o.p.e.'s: the interesting term comes from writing

$$\sigma_{1,1}(x,\bar{x}) = \oint_{C_x} d\bar{y}\epsilon_{ab}\tilde{G}^{-a}(\bar{y})\sigma^b_{1,\frac{1}{2}}(x,\bar{x})\,, \tag{4.3}$$

and using the o.p.e. of $\tilde{G}^{+\pm}(\bar{z})$ with $\tilde{G}^{-\mp}(\bar{w})$. In (4.3) $\sigma^b_{1,\frac{1}{2}} = G^{-b}_{-\frac{1}{2}}\sigma_{\frac{1}{2},\frac{1}{2}}$.

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The simple pole in this o.p.e is given by:

$$\tilde{G}^{+\pm}(\bar{z})\tilde{G}^{-\mp}(\bar{w})|_{\text{simple pole}} = (\tilde{T}(\bar{w}) + \bar{\partial}\tilde{J}_3(\bar{w})/(\bar{z}-\bar{w}\,, \tag{4.4}$$

where $\tilde{T}$ and $\tilde{J}_3$ are respectively the right-moving stress energy tensor and Cartan current of the $SU(2)_R$ R-symmetry. Recalling the o.p.e. of $\tilde{T}$ with $\sigma^a_{1,\frac{1}{2}}(x,\bar{x})$, one sees that this gives a total derivative in $\bar{x}$:

$$\int d^2x\bar{\partial}\sigma^a_{1,\frac{1}{2}}(x,\bar{x})\prod_i \phi_i(z_i,\bar{z}_i) = \sum_i \oint_{C_{z_i}} dx\sigma_{1,\frac{1}{2}}(x,\bar{x})\phi_i(z_i)\prod_{j\neq i}\phi_j(z_j,\bar{z}_j)\,. \tag{4.5}$$

This motivates the following expression for the first order change in the supercharge in the presence of the blowing up mode:

$$\delta\tilde{G}^{+a}_{-\frac{1}{2}}\phi_i(0) = \lambda\oint dx\sigma^a_{1,\frac{1}{2}}(x,\bar{x})\phi_i(0)\,. \tag{4.6}$$

We have considered here the chiral part of the perturbation, the first term in (4.1), however it is easy to see that the antichiral gives the same result (4.6): the reason is that σ and $\bar{\sigma}$ are members of an $SU(2)_R$ doublet,

$$\bar{\sigma}_{\frac{1}{2},\frac{1}{2}} = \oint dzJ^-(z)\oint d\bar{z}\tilde{J}^-(\bar{z})\sigma_{\frac{1}{2},\frac{1}{2}}. \tag{4.7}$$

But then if we move J^- and $\tilde{J}^-$ across G^+ and $\tilde{G}^+$ respectively, we get back G^- and $\tilde{G}^-$, so that, in fact, the two terms in (4.1) are identical .[4]

We see therefore that there is a change in $\tilde{G}^+$ if there is a simple pole $1/x$ in the o.p.e between ϕ and $\sigma_{1,\frac{1}{2}}$. If this is the case, and this will be verified shortly, four states which belong to intermediate multiplets of the (4,4) supersymmetry at the orbifold point should join into long multiplets when turning on the blowing up mode.

Moreover, the action of $\delta\tilde{G}^+_{-\frac{1}{2}}$ on a given field ϕ amounts to fusing $\sigma_{1,\frac{1}{2}}$ with ϕ. Now $\sigma_{1,\frac{1}{2}}$ is in the Z_2 twisted sector and ϕ is in the Z_n twisted sector, so this gives a state in the Z_{n+1} twisted sector when the corresponding two-cycle (i_1,i_2) and n-cycle $(j_1,\dots j_n)$ have one index in common. There will be also a transition from Z_n to Z_{n-1} when the two-cycle is contained in the n-cycle, and this will be the essentially CPT conjugate amplitude involving $\delta\tilde{G}^-_{\frac{1}{2}}$ with the exchange of in- and out-states .[5] Following basically the same steps leading to (4.6), i.e. inserting $\delta\tilde{G}^{-a}_{\frac{1}{2}}$ in the correlator of (4.2) as $\oint_C \bar{z}\tilde{G}^{-a}(\bar{z})$, deforming the contour and using the o.p.e. between $\tilde{G}^{-\pm}(\bar{z})$ and $\tilde{G}^{+\mp}(\bar{w})$, whose relevant part is given by:

$$\tilde{G}^{-\pm}(\bar{z})\tilde{G}^{+\mp}(\bar{w}) \sim \frac{\tilde{T}(\bar{w}) - \bar{\partial}\tilde{J}_3(\bar{w}}{\bar{z}-\bar{w}} - \frac{2\tilde{J}_3(\bar{w})}{(\bar{z}-\bar{w})^2} + \cdots\,, \tag{4.8}$$

[4] We have omitted here, and it will also be understood in the following when not strictly necessary, the $SU(2)_I$ index on the various fields.

[5] Notice that the proper twist fields should correspond to conjugacy classes of S_N, and those corresponding to n-cycles can be obtained from the twist field corresponding to a given n-cycle by averaging over the S_N group orbit.

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one can prove the following equation:

$$\delta\tilde{G}^{-a}_{\frac{1}{2}}\phi_i(0) = \lambda \oint dx\bar{x}\bar{\sigma}^a_{1,\frac{1}{2}}(x,\bar{x})\phi_i(0)\,. \tag{4.9}$$

In this case therefore $\delta\tilde{G}^{-a}_{\frac{1}{2}}$ is determined by the $1/x\bar{x}$ singularity in the o.p.e. between $\bar{\sigma}_{1,\frac{1}{2}}(x,\bar{x})$ and ϕ.

There is however a priori a possibility of transitions from a Z_n-sector state to a Z_{n-1} state which is genuinely different from the previous one. Notice that in the previous discussion we had, on the right-moving sector, chiral states which correspond to a cohomology element with 0 degree on the right.[6] Now, R-charge conservation alone on the right-moving sector would allow a transition via a Z_2 twist field from a Z_n chomology element at degree 0 to a chomology element in the Z_{n-1} of degree 2 (two fermions applied). Similar considerations hold for the conjugate process. We will verify in the following, by an explicit computation that the corresponding amplitude in fact vanishes. Finally, if the incoming two- and n-cycles are disjoint, then one obtains an outgoing cycle which corresponds to a two-particle state and this will not be of interest for us.

The above picture agrees with the result found in section 2, where we have seen that the discrepancy between the supergravity and CFT single-particle partition functions, for states which are right-moving chiral, is of the form $(1-\bar{q}^{\frac{1}{2}}\tilde{y}p)^2 f(p,q,\bar{q}^{\frac{1}{2}}\tilde{y})$, with some given function f. This indeed means that the three supermultiplet's components come from $(n, n+1, n+2)$ twisted sectors, in agreement with the Z_2 twisted character of the modified supercharges discussed above.

We would like now to analyze in more detail the fusion of $\sigma_{1,\frac{1}{2}}$ with the left-moving excited field in the Z_n-twisted sector. We will be interested in a particular class of such fields, namely those obtained by applying an arbitrary string of fractionally moded generators $L_{-1+\frac{k}{n}}$, $G^{-,a}_{-\frac{1}{2}+\frac{l}{n}}$ and $J^-_{\frac{m}{n}}$, where L's and J^-'s are the modes of the stress energy tensor and of the negatively charged R-symmetry current respectively. The integers k, l, m are assumed to be much smaller than n in absolute value, i.e. $|k|, |l|, |m| \ll n$ and have to satisfy orbifold group invariance condition, which demands, in the present case, that the total fractional mode should vanish, i.e. $\sum l_i + \sum k_i + \sum m_i = 0$. This condition corresponds precisely to the level matching condition on the pp-wave side, equation (2.6).

Before discussing the relevant three-point functions, let us consider first the unperturbed two-point functions involving one of these fields and its conjugate. This correlator will involve multiple insertions of an even number of T's, of pairs of G^+'s and G^-'s and of J^+'s and J^-'s. The particular mode insertion can be then obtained by a contour integration around the positions of the chiral and conjugate anti-chiral external fields. The following observation will be useful when we will discuss the perturbed two point functions: in the large-N limit we are interested in, the leading contribution to this correlator

[6] The single particle cohomology elements of $(T^4)^N/S_N$ from a given Z_n twisted sector inherit the Dolbeault degrees (p,q) of the original T^4, with $p,q = 0,1,2$. In the CFT, $p=0$ ($q=0$) corresponds to the basic Z_n twist field on the left (right), whereas $p=1,2$ ($q=1,2$) correspond to applying one or two left- (right-) moving invariant fermions, respectively, with positive R-charge. This increses the dimension and the charge by the same amount so that the resulting field is still chiral primary.

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comes from "diagonal" contractions, where each of the above generators is contracted with a conjugate generator. The reason for this comes from the fact that in the o.p.e. of two conjugate operators one finds schematically T, $T+\partial J_3$ or J_3. When bringing these operators near the chiral primary in the Z_n twisted sector, they will give a contribution of order n, because they will measure the dimension or the charge (or the sum of the two) of the corresponding field, which is $(n-1)/2$. Off-diagonal contractions will be suppressed with respect to the diagonal ones by powers of n. This argument also says that in order to have two-point functions of order 1, we should normalize the generators with a $1/\sqrt{n}$ factor.

This observation facilitates the analysis of the three point function, i.e. the two point function involving the blowing up mode to first order, as far as the leading contribution for large n is concerned. In this case of course the presence of the blowing up mode forces the two external excited states to be non-diagonal. However, from the previous argument, the leading contribution will arise when all the generators inserted will be contracted diagonally, apart from a pair which has to connect through $\sigma_{1,\frac{1}{2}}$. The upshot of this argument is that one can concentrate on the part of the correlator which involves a "flip" of the generators via the interaction $\sigma_{1,\frac{1}{2}}$. Charge conservation then restricts the consideration to two type of correlators:[7]

$$\begin{aligned} C_1(u) &= \langle n+1|J^+_{-\frac{k}{n+1}}\sigma_{1,\frac{1}{2}}(u,\bar{u})G^-_{-\frac{1}{2}+\frac{k}{n}}|n\rangle\,, \\ C_2(u) &= \langle n+1|G^+_{+\frac{1}{2}-\frac{k}{n+1}}\sigma_{1,\frac{1}{2}}(u,\bar{u})L_{-1+\frac{k}{n}}|n\rangle\,, \end{aligned} \tag{4.10}$$

where $|n\rangle$ and $|n+1\rangle$ denote ground states in the Z_n and Z_{n+1} twisted sector respectively.

Before turning to a more detailed computation, let us observe that the u, $\bar{u}$ dependence of the three-point functions in (4.10) (or rather of the full three-point function, involving physical states obeying level-matching condition) is fixed by the conformal dimensions of the three fields, and therefore it is of the form $c_{n,n+1}/u$, where $c_{n,n+1}$ is a structure constant, and we have assumed that the external twist fields are at 0 (Z_n) and ∞ (Z_{n+1}). Therefore the non-trivial task for our purposes is the determination of the leading n dependence of the structure constants, and this will give us the action of the modified right-moving supercharges on the excited states we are interested in.

In order to perform the computation of the above correlators it is convenient to introduce a covering map from the plane t, on which the correlator is single valued, the original plane z on which it has the appropriate monodromies. This map can be chosen to be:

$$z = t^n(t-t_0)\,, \tag{4.11}$$

with some arbitrary parameter t_0. This is a branched covering, with branch point of order n at $t=0$, of order $n+1$ at $t=\infty$ and of order 2 at $x=nt_0/(n+1)$, where the Z_2 twist fields is located. This map thus realizes the required monodromies. Denoting with u the value of z at $t=x$, we have

$$u = z(x) = -x^{n+1}/n\,. \tag{4.12}$$

[7] The following correlators are understood as part of a correlator which involve states satisfying the orbifold group invariance condition.

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In terms of the t coordinate the scalar correlator is simply

$$\partial X(z)\partial\bar{X}(w) = \frac{t'(z)t'(w)}{(t(z)-t(w))^2}\,. \tag{4.13}$$

It is also convenient to bosonize the two complex fermions $\Psi_I^{+\pm}$ by introducing two canonically normalized scalar fields H_1^I and H_2^I as follows

$$\begin{aligned}\Psi_I^{++} &= \exp\left(iH_1^I + iH_2^I\right)/\sqrt{2}\\ \Psi_I^{+-} &= \exp\left(iH_1^I - iH_2^I\right)/\sqrt{2}\,,\end{aligned} \tag{4.14}$$

which both carry charge 1/2 with respect to $J_3 = (\sum_I H_1^I)/\sqrt{2}$. Obvious expressions, with signs flipped, hold for the conjugate fields Ψ^{--} and Ψ^{-+}. The index $I = 1, \ldots, N$ refers to the I-th field copy. As for the twist field in the Z_n twisted sector, it carries charge $(n-1)/2$ with respect to J_3 and therefore its H_1 dependence is given by

$$\exp\left(i\frac{(n-1)\sum H_1^I}{\sqrt{2}n}\right)\,, \tag{4.15}$$

whereas it does not depend on H_2. The total dimension of the twist field is given by the sum of the dimension of the charged component, $(n-1)^2/4n$, and the dimension of the neutral one, $(n^2-1)/4n$, which gives a total of $(n-1)/2$, in agreement with its chiral primary nature. Similar expressions hold for the right-moving fields.

Let us consider first the three point function C_1. It is convenient to perform the calculation on the covering t-plane, where it is easy to find the following expression:

$$\begin{aligned}C_1 = \frac{1}{(n+1)x^{n-1}}\oint_\infty dt_2 \oint_0 dt_1 (t_2)^{-\frac{kn}{n+1}}(t_2-t_0)^{-\frac{k}{n+1}}(t_1)^k(t_1-t_0)^{\frac{k}{n}} \times\\ \times\left(\frac{t_2}{t_1}\right)^{n-1}\frac{1}{(t_2-t_1)(t_1-x)^3}\,,\end{aligned} \tag{4.16}$$

where the contour integrations with the corresponding k-dependent powers project on the given fractional modes. The latter t_2 and t_1 dependent factors are, in terms of the z coordinate, just $z_2^{-\frac{k}{n+1}}$ and $z_1^{\frac{k}{n}}$ respectively. It is convenient to rescale t_1 and t_2 by x, after which we will get, for n large, an overall factor $1/x^{n+1}$ in (4.16), together with the replacement of t_0 by $n/(n+1)$, which is equal to 1 in the large-N limit. The prefactor $1/x^{n+1}$ was to be expected on dimensional grounds, since it gives a simple pole in the physical coordinate u, in view of the relation (4.12), which gives also an additional factor $1/n$ in the correlator. It remains to evaluate the contour integrals in (4.16), which is most easily done by doing a series expansion of the integrand. More details are given in appendix A, but is easy to see that the leading contribution is of order kn for n large. To estimate the overall behavior of the complete correlator, including "spectator" generators, one should take into account that each generator comes with a normalization factor $1/\sqrt{n}$, as already mentioned, so that the contribution of the diagonal contractions is of order 1. Furthermore, by doing the relevant combinatorics, one can show that the three point function of normalized ground states $\langle n+1|\sigma_{\frac{1}{2},\frac{1}{2}}|n\rangle$ goes like n for large n. So, collecting all the factors of n, we see that the full three point function behaves like k/n for large-N.

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The above result could have been obtained perhaps more transparently by using a superconformal Ward identity which relates C_1 to the correlator

$$C_1' = \langle n+1|J^+_{-\frac{k}{n+1}}\sigma_{\frac{1}{2},\frac{1}{2}}(u,\bar{u})J^-_{-\frac{k}{n}}|n\rangle. \tag{4.17}$$

The Ward identity is obtained by rewriting $G^-_{-\frac{1}{2}+\frac{k}{n}}|n\rangle$ in C_1 as $G^+_{-\frac{1}{2}}$ applied to $J^-_{\frac{k}{n}}|n\rangle$ and doing the contour deformation we used previously. The only contribution arises when $G^+_{-\frac{1}{2}}$ hits $\sigma_{1,\frac{1}{2}}(u,\bar{u})$, which gives $L_{-1}\sigma_{\frac{1}{2},\frac{1}{2}}(u,\bar{u})$. So we get as a result of these manipulations that $C_1(u) = \partial_u C_1'(u)$. On the other hand, on dimensional grounds, the u dependence of C_1' is $u^{\frac{k}{n(n+1)}}$, so that the u derivative produces a suppression factor $1/n^2$. The n dependence of C_1' itself is quickly estimated in the following way: to the leading order in $1/n$ the fractional modes of J^+ and J^- are opposite to each other, so that when we bring say J^+ against $J^-|n\rangle$ we get $J^3_0|n\rangle$, that is a factor proportional to n. Taking into account the normalization of the currents, we see that C_1' is of order 1. Therefore again we see that C_1 is indeed of order $1/n$ in agreement with the previous analysis, if we take into account the normalization of the three twist field correlator which goes like n.

Let us come now to the other basic correlator C_2: again using SCFT Ward identities we can relate it to C_1 plus another correlator. This is done by writing

$$L_{-1+\frac{k}{n}}|n\rangle = \left(G^+_{-\frac{1}{2}}G^-_{-\frac{1}{2}+\frac{k}{n}} + \frac{k}{n}J^3_{-1+\frac{k}{n}}\right)|n\rangle\,. \tag{4.18}$$

The first term in (4.18) can be shown, after bringing G^+ against the other operators, to be equal to:

$$C_1 + \langle n+1|G^+_{\frac{1}{2}-\frac{k}{n+1}}\partial_u\sigma_{\frac{1}{2},\frac{1}{2}}(u,\bar{u})G^-_{-\frac{1}{2}+\frac{k}{n}}|n\rangle\,, \tag{4.19}$$

where C_1 arises when $G^+_{-\frac{1}{2}}$ acts on $\langle n+1|G^+_{\frac{1}{2}-\frac{k}{n+1}}$ and the second term when we commute G^+ with $\sigma_{1,\frac{1}{2}}$.

The term involving J^3 in (4.18) can be shown explicitly, using the procedure explained in appendix A, to be suppressed by a power of n with respect to C_1. We are left therefore with the second term in (4.19). It is easy to see that this term is suppressed too in the large-N limit compared to C_1: the reason is that all the n countings go in the same way as for the J^+, J^- correlator of the previous paragraph, but there is an additional $1/n$ suppression factor coming from the jacobians appearing when relating the t coordinates to the z coordinates. They are there in this case (as well as in (4.16), giving the prefactor $1/(n+1)$) because the supercurrents which are being contour integrated have weight $3/2$, whereas they are absent in the case of the two currents J^+ and J^-, which have weight 1, and therefore their contour integral is invariant under coordinate changes. Morew details are given in appendix A.

Let us discuss now amplitudes which involve a change in the cohomology's degree, and show, as anticipated before, that they do not occur in the present situation. To this purpose it is sufficient to consider the right moving sector of the amplitude:

$$\langle n+1|\tilde{G}^-_{\frac{1}{2}}\sigma_{1,1}\tilde{\Psi}^{++}_{-\frac{1}{2}}\tilde{\Psi}^{+-}_{-\frac{1}{2}}|n\rangle\,. \tag{4.20}$$

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One can rewrite this correlator in the following way:

$$\langle n+1|\oint_\infty dz_3 z_3 \tilde{G}^-(z_3)\oint_u dz \tilde{G}^-(z)\sigma_{1,\frac{1}{2}}(u)\oint_0 \frac{dz_1}{z_1}\tilde{\Psi}^{++}(z_1)\oint_0 \frac{dz_2}{z_2}\tilde{\Psi}^{+-}(z_2)|n\rangle\,. \tag{4.21}$$

This expression is computed using the t coordinates, using the map $z_i = t_i^n(t_i - t_0)$ and keeping track of the variuos jacobians as explained already. One can perform then first the t, t_1 and t_2 contour integrations, but then one finds that the final t_3 contour integral vanishes, being of the form $\oint_\infty dt_3(t_3 - t_0)/(t_3 - x)^3$.

We can now use the above results to compute, to the first non-trivial order, the change in the right-moving conformal dimensions of the left-excited states under discussion. The starting point is the asnti-commutation relation:

$$\left\{\tilde{G}^-_{+\frac{1}{2}}, \tilde{G}^+_{-\frac{1}{2}}\right\} = \tilde{L}_0 - \tilde{J}^3_0. \tag{4.22}$$

Here $\tilde{G}^\pm_{\pm\frac{1}{2}}$ is actually $\tilde{G}^{(0)\pm}_{\pm\frac{1}{2}} + \delta\tilde{G}^\pm_{\pm\frac{1}{2}}$, with $\tilde{G}^{(0)\pm}$ the zeroth order supercharge.

We can take now the matrix element of (4.22) between any of the (normalized) excited states proportional $J^-_{-\frac{k}{n}}|n\rangle$, $G^-_{-\frac{1}{2}+\frac{k}{n}}|n\rangle$ or $L_{-1+\frac{k}{n}}|n\rangle$ and its conjugate. Notice that these states are related to each other by supersymmetry. Consider for example $|\psi_n\rangle \sim J^-_{-\frac{k}{n}}|n\rangle$. From equation (4.10) we have:

$$\delta\tilde{L}_0 - \tilde{j} = \langle\psi_n|\delta\tilde{G}^+_{-\frac{1}{2}}|\psi'_{n-1}\rangle\langle\psi'_{n-1}|\delta\tilde{G}^-_{+\frac{1}{2}}|\psi_n\rangle = \lambda^2|c_{n-1,n}|^2\,. \tag{4.23}$$

Here we have used the fact that the zeroth order $\tilde{G}^{(0)}+_{-\frac{1}{2}}$ annihilates $|\psi_n\rangle$, so that there is no need to consider the second order change in $\tilde{G}^+_{-\frac{1}{2}}$. The excited state $|\psi'_{n-1}\rangle$, from the Z_{n-1} twisted sector, is proportional to $G^-_{-\frac{1}{2}+\frac{k}{n-1}}|n-1\rangle$. From (4.23), we see that indeed the leading order change in the right-moving conformal dimensions for the above states is of order $(k/n)^2$ for a given mode k. What remains to do is to fix its N dependence. Recall that the external states, from the Z_n twisted sector, are canonically normalized, however we have to fix the normalization of the blowing up mode. If we assume that the coupling constant λ equals the effective 6-dimensional string coupling $g_6 = g_s\sqrt{\frac{Q_5}{Q_1}}$, where g_s is the string coupling,[8] and the Z_2 chiral primary σ is actually obtained by summing over the two-cycles (i,j), i.e. it is of the form $\sigma = \sum_{i<j}^N \sigma_{ij}$, then the three point function above is of order $\sqrt{N} = \sqrt{Q_1Q_5}$, since the canonically normalized Z_2 twist field is σ/N and the three-point function of canonically normalized twist fields goes like $1/\sqrt{N}$. Collecting all the factors we get a correction of the form

$$\frac{1}{2}(g_s)^2(Q_5)^2\left(\frac{k}{n}\right)^2 \tag{4.24}$$

for $\Delta = L_0 + \bar{L}_0$, which is in agreement with the first order expansion of the pp-wave hamiltonian for massive oscillators, when translated into CFT units.

[8] Independent arguments supporting this identification have been given in [15].

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We now discuss another class of states, which should correspond to T^4 massless oscillator states in the light-cone hamiltonian on the pp-wave. These are non-chiral on both left- and right-moving sectors, and are obtained by applying fractional modes of the $\mathrm{U}(1)^4_L \times \mathrm{U}(1)^4_R$ super-current algebra, generated by the four free bosons and fermions, to the ground state in the Z_n twisted sector. Level-matching requires that $L_0 - \tilde{L}_0 = 0 \bmod n$. Neither of the unperturbed supercharges $G^+_{-\frac{1}{2}}$ and $\tilde{G}^+_{-\frac{1}{2}}$ annihilates these states, and to zeroth order in the blowing up mode both $\tilde{L}_0 - \tilde{J}^3_0$ and $L_0 - J^3_0$ are of the form k/n, with $k \ll n$. The question is what happens when we switch on the blowing up mode. In general their conformal dimensions will change, but in order for the change to agree with the pp-wave string spectrum it should happen that, to any order in in the blowing up mode coupling constant, the change should be proportional to k/n. In this case the effect of turning on the blowing up mode will be to renormalize k/n by some finite amount depending on its coupling constant. We will not be able to prove this, but we will perform structure constant calculations similar to those described previously, which will confirm the above picture, giving indeed structure constants of order $\sqrt{k/n}$.

Before turning to the calculation of the relevant three point functions, let us fix the correct normalization of the fractionally moded bosonic and fermionic oscillators. We have, using the covering coordinates t's, for the left-moving bosons:

$$\langle n|\bar{\alpha}_{\frac{k}{n}}\alpha_{-\frac{k}{n}}|n\rangle = \oint_0 dt_1 \oint_\infty dt_2 \left(\frac{t_2}{t_1}\right)^k \frac{1}{(t_1-t_2)^2} = k\,, \tag{4.25}$$

whereas for the left-moving fermions:

$$\langle n|\Psi^{+\mp}_{-\frac{1}{2}-\frac{k}{n}}\Psi^{-\pm}_{\frac{1}{2}+\frac{k}{n}}|n\rangle = \oint_0 \frac{dt_1}{\sqrt{t_1'}} \oint_\infty \frac{dt_2}{\sqrt{t_2'}} \frac{1}{(t_1-t_2)} \frac{{t_2}^{n-1}}{t_1^n} \left(\frac{t_2}{t_1}\right)^k = n\,. \tag{4.26}$$

Similar expressions hold for the right-movers.

Thus we should rescale each bosonic (fermionic) oscillator by $1/\sqrt{k}$ ($1/\sqrt{n}$) when constructing normalized excited states.

The arguments of the first part of this section concerning the modification of G^+, $\tilde{G}^+$ in the presence of the blowing up mode go through also for this class of states, the only difference being that now there is a non-trivial zeroth-order action of G^+, $\tilde{G}^+$ as well. In particular, at first order in the perturbation, we should compute three-point functions involving external excited states and an insertion of $\sigma_{1,\frac{1}{2}}$ or $\sigma_{\frac{1}{2},1}$. Let us consider the first case. In order for the three point function to be non-vanishing, the two external states should contain operators acting on Z_n and Z_{n+1} ground states respectively , which, in the right-moving sector, are conjugate of each other whereas in the left-moving sector they are conjugate apart from a single flip due to the presence of $\sigma_{1,\frac{1}{2}}$. R-charge conservation then requires in the left-moving sector there should be a flip from a bosonic oscillator to a fermionic oscillator carrying $+1/2$ R-charge. The basic correlators we need to consider are therefore:[9]

$$\begin{aligned} D_1(u) &= \langle n+1|\Psi^+_{-\frac{1}{2}+\frac{k}{n+1}}\tilde{\alpha}_{\frac{k}{n+1}}\sigma_{1,\frac{1}{2}}(u,\bar{u})\alpha_{-\frac{k}{n}}\tilde{\bar{\alpha}}_{-\frac{k}{n}}|n\rangle\,, \\ D_2(u) &= \langle n+1|\Psi^+_{-\frac{1}{2}+\frac{k}{n+1}}\tilde{\Psi}^+_{-\frac{1}{2}+\frac{k}{n+1}}\sigma_{1,\frac{1}{2}}(u,\bar{u})\alpha_{-\frac{k}{n}}\tilde{\Psi}^-_{\frac{1}{2}-\frac{k}{n}}|n\rangle\,. \end{aligned} \tag{4.27}$$

[9] We omit here the $\pm$ index corresponding to the direction 2 in (4.14)

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These correlators can be computed along the lines we followed to evaluate $C_{1,2}$. Let su first connsider D_1: one starts from the correlator:

$$\langle n+1|\Psi^+(t_2)\bar{\partial}X(\bar{t}_2)\sigma_{1,\frac{1}{2}}(u,\bar{u})\partial X(t_1)\bar{\partial}\bar{X}(\bar{t}_1)|n\rangle\,, \tag{4.28}$$

and then projects onto the given modes by contour integrating with the factors $z_1^{\frac{k}{n}}$ and $z_2^{-\frac{k}{n+1}}$ as before. It is easy to see that the resulting expression , when expressed in terms of the physical coordinate u goes like k^2/nu. Taking into account the normalization factors discussed above, giving a factor $1/k^{\frac{3}{2}}\sqrt{n}$ and the three-ground state correlator, which goes like n, we see that the resulting structure constant $d^1_{n,n+1}$ in $D_1 \sim d^1_{n,n+1}/u$ goes like $\sqrt{\frac{k}{n}}$.

One can similarly evaluate $d^2_{n,n+1}$ in D_2, starting with

$$\langle n+1|\Psi^+(t_2)\bar{\partial}\tilde{\Psi}^+(\bar{t}_2)\sigma_{1,\frac{1}{2}}(u,\bar{u})\partial X(t_1)\Psi^-(\bar{t}_1)|n\rangle\,, \tag{4.29}$$

and proceeds in the same way. In this case the correlator goes like k/u, so that taking into account the normalization factors, which give now $1/n^{\frac{3}{2}}\sqrt{k}$, and usual factor of n from the ground states correlator, gives for the structure constant $d^2_{n,n+1} \sim \sqrt{\frac{k}{n}}$.

In this case, unlike in the previous case, the first order structure constants above do not allow to compute the full change in the conformal dimensions, since the zeroth order supercharges do not annihilate the states and we need also the second order expression. So, what is required is really a full fledged four point function calculation. Nevertheless we believe that the results above give a good indication that indeed the variations of the conformal dimensions at arbitrary orders in the blowing up mode are proportional to k/n.

5. Discussion

In the last section we saw that the first order correction to the dimension matches with the prediction from the pp-wave side. Now we discuss the higher order corrections for the states corresponding to the massive oscillators.

The relevant set of states are the excitations of the Z_n twisted chiral-chiral state by applying fractional moded $J^-_{k/n}$, $G^-_{-1/2+k/n}$ and $L_{-1+k/n}$ operators. Let us start from the state $|\psi_n\rangle \equiv N_n J^-_{-\frac{k}{n}}|n\rangle$ where N_n is the normalization constant so that $|\psi_n\rangle$ has unit norm. This is not a physical state; there must be multiple insertions of fractional modes so that the total level is an integer (or half integer for fermionic states). However we have seen in the first order analysis that in the large-N limit, it is sufficient to consider the interaction of only one of the insertions, the remaining ones are spectators and contract diagonally. $(j,\tilde{j})$ charge of $|\psi_n\rangle$ is $(\frac{n-3}{2},\frac{n-1}{2})$. Using methods given in the last section one can see that $\tilde{G}^+_{-1/2}$ and $G^-_{1/2}$ annihilates this state at least to the first order in the perturbation. Moreover $\tilde{G}^-_{1/2}|\psi_n\rangle$ is proportional to $G^+_{-1/2}|\psi_{n-1}\rangle$. It seems natural to assume that to all orders in the perturbation, under the action of $\mathcal{N}=(4,4)$ generators, the set of states under consideration are mapped to each other (in large-N limit). If this is so then the above first order results must be true to all orders. This is seen simply by taking into account the left and right U(1) charges of the states.

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Now using the supersymmetry algebra we find that the norm square of the state $\tilde{G}^-_{1/2}|\psi_n\rangle$ is

$$\left\|\tilde{G}^-_{1/2}|\psi_n\rangle\right\|^2 = \gamma\langle\psi_n|\psi_n\rangle \tag{5.1}$$

where $\gamma = \tilde{L}_0 - \tilde{j}$ eigenvalue of $|\psi_n\rangle$.

On the other hand, we have seen from the three point computation given in the last section, that to the first order in perturbation:

$$\langle\psi_{n-1}|G^{-a}_{1/2}\tilde{G}^{-b}_{1/2}|\psi_n\rangle = \epsilon^{ab}\frac{\lambda k\sqrt{N}}{2n}\,. \tag{5.2}$$

If we assume that this equation is true to all orders in λ/n, then, using the fact that the norm square of the state $G^+_{-1/2}|\psi_{n-1}\rangle$ is $L_0 - j$ eigenvalue of $|\psi_{n-1}\rangle$ which is equal to $\gamma+1$ (note that the correction to L_0 and $\tilde{L}_0$ must be equal), we conclude that:

$$\gamma = \frac{1}{\gamma+1}\frac{\lambda^2k^2N}{4n^2}\,. \tag{5.3}$$

The solution to this equation is

$$\gamma = \frac{1}{2}\left(\sqrt{1+\frac{\lambda^2k^2N}{n^2}} - 1\right)\,. \tag{5.4}$$

Identifying $n/\lambda\sqrt{N}$ with p^+ and the fact that the correction to Δ is twice the correction to $\tilde{L}_0$ and hence 2γ, we find complete agreement with the pp-wave spectrum.

This incidentally also proves the formula for $\tilde{G}$ given in (3.12). Indeed it follows that

$$\tilde{G}^-_{1/2}|\psi_n\rangle = \frac{\lambda k\sqrt{N}}{n}\frac{1}{\sqrt{\gamma+1}}|\psi'_{n-1}\rangle \tag{5.5}$$

where $|\psi'_{n-1}\rangle = \frac{1}{\sqrt{\gamma+1}}G^+_{-1/2}|\psi_{n-1}\rangle$ has unit norm. Using the value of γ and the identification of p^+ as above, we find that:

$$\tilde{G}^-_{1/2}|\psi_n\rangle = \sqrt{\frac{\omega_k - p^+}{p^+}}\,|\psi'_{n-1}\rangle \tag{5.6}$$

which is exactly the relevant term appearing in the expression of $\tilde{G}$ from the pp-wave side in (3.12) .

The exact correspondence with the pp-wave therefore rests on the validity of equation (5.2) to all orders. Since $G^-_{1/2}$ annihilates $|\psi_n\rangle$, this is equivalent to the statement that

$$\left\{\tilde{G}^{-a}_{1/2}G^{-b}_{1/2}\right\}|\psi_n\rangle = \epsilon^{ab}\lambda\int dz\partial_z\left(z\bar{z}\bar{\sigma}_{\frac{1}{2},\frac{1}{2}}\right)|\psi_n\rangle \tag{5.7}$$

where the integration contour encircles the insertion of the state $|\psi_n\rangle$. This equation is the analog of equation (3.11) appearing in the pp-wave string, provided the operator on the right-hand side is identified with P_σ/p^+. The latter vanishes on the physical states though not on the individual oscillators. This is also the case with the operator $\int dz\partial_z(z\bar{z}\bar{\sigma}_{\frac{1}{2},\frac{1}{2}})$

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appearing above: physical states consisting of multiple insertions satisfying $\sum k_i = 0$ have integer (or half-integer for fermionic states) unperturbed dimensions and therefore are local with respect to $\bar{\sigma}_{\frac{1}{2},\frac{1}{2}}$ and thus are annihilated by this operator. This is not the case with the individual building blocks of such states. Indeed the individual operator insertion carries fractional dimension k/n, while the outgoing state carries dimension $k/(n-1)$. The difference of the dimensions being of order k/n^2, gives rise to the OPE:

$$z\bar{z}\bar{\sigma}_{\frac{1}{2},\frac{1}{2}}(z,\bar{z})|\psi_n\rangle \sim n.\frac{k}{n^2}\log z|\psi_{n-1}\rangle \tag{5.8}$$

resulting in equation (5.2). Here the first factor of n on the right hand side comes because there are n channels by which $\bar{\sigma}$ can take Z_n to Z_{n-1} twisted sector.

Analogous discussion for the states corresponding to the massless pp-wave modes is more complicated. Consider the state $|\phi_n\rangle \sim \psi^-_{1/2-k/n}\tilde{\psi}^+_{-1/2-k/n}|n\rangle$ for $k\rangle 0$. Under the assumption that the set of states corresponding to all the massless pp-wave modes close under the action of the symmetry algebra, we can again conclude that $G^-_{1/2}$ and $\tilde{G}^+_{-1/2}$ annihilates this state. If we further assume that $\tilde{G}^-_{1/2}|\phi_n\rangle$ is proportional to $G^+_{-1/2}|\phi_{n-1}\rangle$ then a discussion similar to above using the eq. (5.2) would now imply that $\gamma^2 = \lambda\sqrt{N}k/n$ in agreement with the pp-wave prediction. However the problem is that there are two different states having the same left and right U(1) charges as those of $\tilde{G}^-_{1/2}|\phi_n\rangle$ and $G^+_{-1/2}|\phi_{n-1}\rangle$. Therefore the consideration of charges is not sufficient to conclude that the latter two states are proportional to each other.

We conclude by noting that the issue of higher order corrections is an important open question. The discussion we have given above indicates that the crucial identity that one needs to prove is equation (5.2). This equation suggests that there is an extension of the $\mathcal{N} = (4,4)$ algebra which collapses to the usual one on the physical states. This is certainly the case on the pp-wave side and to the first order in the perturbation also in the boundary CFT. It is also interesting to note that equation (5.2) plays the role of the equation of motion used in [23] to derive an all order formula for the anomalous dimensions in the $\mathcal{N} = 4$ Yang-Mills theory.

A. The three point function

In this appendix we give some details of the computation of the correlation functions in section... The main object we have analyzed is the three point function denoted by C_1, which is given by:

$$\begin{aligned} C_1 = \oint_\infty dt_2 \oint_0 dt_1 \big(z_2(t_2)\big)^{-\frac{k}{n+1}} \big(z_1(t_1)\big)^{\frac{k}{n}} \sqrt{t_1'} \times \\ \times \langle n+1|J^+(t_2) \oint_x dt\sqrt{t'} G^-(t)\sigma_{\frac{1}{2},\frac{1}{2}}(x,\bar{x})G^-(t_1)|n\rangle\,, \end{aligned} \tag{A.1}$$

where the prime on t (t_1) means that it is differentiated with respect to z (z_1), giving:

$$t' = \frac{1}{t^{n-1}(t-x)(n+1)}\,, \tag{A.2}$$

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with similar expression for t_1, with $x = nt_0/(n+1)$. As explained before, the presence of the square roots for the contour integrations of the supercurrents is due to the fact that they have dimension 3/2. Using the scalar correlator (4.13) and the bosonization expressions (4.14) and (4.15) one can easily reproduce (4.16). After rescaling t_1 and t_2 by x one is left, in the limit of large-N, with the integral:

$$C_1 = \frac{1}{(n^2)u} \oint_\infty dt_2 \oint_0 dt_1 (t_2)^{-\frac{kn}{n+1}} (t_2-1)^{-\frac{k}{n+1}} (t_1)^k (t_1-1)^{\frac{k}{n}} \times \\ \times \left(\frac{t_2}{t_1}\right)^{n-1} \frac{1}{(t_2-t_1)(t_1-1)^3} \,. \tag{A.3}$$

It is convenient to perform first the t_2 integral and then the t_1 integral, after expanding the integrand in power series. In this way one gets:

$$C_1 = \frac{1}{n^2 u} \sum_{j=1}^{n+k-1} c(k/n)_j c(-k/n-3)_{(j-1)} \,, \tag{A.4}$$

where the coefficients $c(\alpha)_i$ are defined to be:

$$c(\alpha)_i = \frac{\Gamma(i-\alpha)}{\Gamma(-\alpha)\Gamma(i+1)} \tag{A.5}$$

which, for small α behave like $c(\alpha)_i \to -\alpha/i$, for $i \neq 0$, whereas $c(\alpha)_0 = 1$. Using the above definition of the c_i's, in particular the fact that, since $j \rangle 0$, $c(k/n)_j$ goes like $-1/kn$, one sees that the above sum for large n is proportional to:

$$\sum_{j=1}^{n} \frac{k}{nj} j(j+1) \sim \frac{1}{2} nk \,. \tag{A.6}$$

We thus see that the structure constant $c_{n,n+1}$ is of order k/n.

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